A MODEL OF OPTIMAL BRAIN DRAIN[1]

Nguyen Duc Thanh

National Economics University, Hanoi, Vietnam

National Graduate Institute for Policy Studies, Tokyo, Japan

Email: d0311@stu.grips.ac.jp

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Preliminary - Comments Welcome

Abstract

This paper is a contribution to a new line of theory which argues that brain drain is not always a bad thing to the source country. First, it enriches the methodology by solving the problem with assumption on workers' heterogeneous talents. Second, in contrast to the previous literature, this paper shows that positive effect of brain drain may never take place under some certain conditions, and proposes to call it *"brain drain trap."* Third, if there is positive effect, there exists a unique value of emigration probability to maximize the gain from brain drain - or the *"optimal brain drain"* value. Relevant policies on education and emigration for the source country are then suggested.

Keywords: Brain drain; Human capital formation; Migration; Small-open economy

JEL classification: F22, J24, F43

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INTRODUCTION

The term “brain drain” was popularized after WW II when there was a huge number of leading scientists immigrating to United State from Western Europe, Canada and Soviet Union (Rapoport 2002). However, the causes and consequences of brain drain led to debates and resolutions in the United Nations only as early as 1967, concerning the argument that the poor countries lost their most talented people to the rich countries (Lowell 2002b). During the 1970s, many economists paid attention to the issue, creating the first wave of “brain drain” economics. Notable economist Jagdish Bhagwati, among others, may be the most influencing figure in the debates. Economists during the period shared, more or less, a consensus that brain drain is a zero-sum game, in which the rich nations gain on the loss of the poor nations. (Bhagwati and Hamada 1974, Bhagwati 1976, Bhagwati and Partington 1976, Hamada 1977, Bhagwati 1979a, 1979b, and later Kowk and Leland 1982). This first wave seemed to fade away with the decline of the “first generation” of development economics in the late 1970s[2]. It must wait for almost two decades to see the second wave taking place, following the raise of “new” growth paradigm, in which human capital was realized as an important engine of economic growth.

Mountford (1997) for the first time argues that brain drain is not always a “curse” to poor countries, if not an effective way to escape from the “poverty trap”. His argument is that people in a poor country may have stronger motivation to get more skills if they see some probability of emigrating to a rich country, where they can earn more with the same level of human capital. This line of thinking has been developed theoretically (Vidal 1998, Stark et al. 1997), and empirically (Beine et al. 2001) . As a result, a new generation of brain drain policy is introduced (Stark et al 1998, Stark and Wang 2002, Stark 2004).

This paper is a contribution to this line of theory. It develops a model reconfirming that brain drain is not always a bad thing to the source country. But it differs from previous literature in some aspects. First, the paper loosens the homogeneous worker assumption. Second, in contrast to the previous studies, this paper shows that positive effect of brain drain is not inevitable: under some conditions, this effect never occurs. Third, if there is positive effect, there exists a unique value of emigration probability to maximize the gain from brain drain - the optimal brain drain.

The next section introduces the model, which includes 3 parts: assumption settings, model’s solution, and policy implications withdrawn from the theoretical results. The final section is conclusion, which provides a summary and suggestions for further studies.

THE MODEL

Assumptions

Workers’ talent (τ):

Following Lucas (1988), consider a small economy including *N* workers with different degrees of talent. A worker’s talent τ_i follows a given probability distribution function, $p(\tau)$. This means, the probability of a worker with degree of talent τ_i is $p(\tau_i)$, or the number of people

[2] See Barro and Sala-i-Martin (2004: 16-21) for a brief review of phases in development of growth theory.

with talent τ_i is: $n_i = Np(\tau_i)$. The following conditions hold: $\lim_{\tau \to 0} p(\tau) = 0$ and $\lim_{\tau \to \infty} p(\tau) = 0$ (Figure 1).

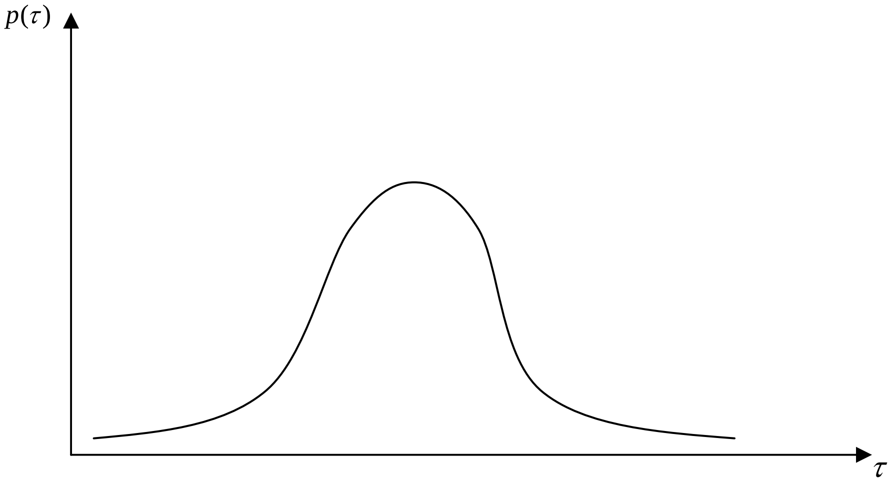

Figure 1. Probability distribution function of a worker's talent

As a continuous probability distribution function, this condition holds: $\int_0^\infty p(\tau) d\tau = 1$

Cost of education (c) and human capital formation (h):

Workers work and at the same time choose to invest on their own human capital. The human capital investment expenditure is c_i. If one invests c_i, she will accumulate a stock of human capital h_i:

$$h_i = h_i(c_i, \tau_i) .$$

$h(c, \tau)$ may be called the human capital formation function. This function shows that the human capital accumulated depends on the worker's talent and her human investment expenditure. In principle, she can receive more education from school or more skills from leaning-by-doing or from any source, but these activities are costly in terms of real resource, which are counted in c_i.

In general, h holds the following properties:

$$\frac{\partial h}{\partial c} > 0; \qquad \frac{\partial^2 h}{\partial c^2} < 0 ; \qquad \frac{\partial h}{\partial \tau} > 0$$

From these properties, one can obtain the substitution rate of investment expenditure and talent:

$$\frac{dc}{d\tau} = -\frac{\frac{\partial h}{\partial \tau}}{\frac{\partial h}{\partial c}} < 0;$$

This implies that to attain a same amount of human capital, the more talented the worker is, the less real resource she has to sacrifice.

In this paper, for simplicity I assume the human capital formation function as:

$$h_i = c_i^{\alpha}\tau_i \qquad (0 < \alpha < 1)$$

Worker's total income (TU):

The compensation for worker's labor is assumed equal to her level of human capital stock: $U_i = h_i$. That means, her life income is:

$$TU_i = -c_i + h_i(c_i, \tau_i).$$

Probability of emigration (π):

Each worker has chance to emigrate to another country where the marginal human capital product is higher. Therefore, at any level of human capital stock, the successfully emigrating worker will receive an income ω times higher than the same worker working domestically, or:

$$U_{i(migrate)} = \omega U_i \qquad (\omega > 1).$$

Suppose the probability of success is $\pi = \pi(h)$. π can be an increasing, decreasing, or constant function of *h*, depending on migration policies. In this paper, π is assumed to be constant.[3]

Objective function:

It is assumed that each worker decides how much real resource to invest in her human capital to maximize her expected life income. Or:

$$\underset{(c)}{Max}(TU_i) = -c_i + E(U_i).$$

Aggregate human capital stock (H)*:*

Having solved the maximization problem of her own, the worker at given talent τ_i will choose to invest $c_i^*(\tau_i)$ in her human capital, therefore she possesses a level $h_i^*(\tau_i)$ of human capital. Thus, total human capital stock of the economy H is the sum of all $h_i^*(\tau_i)$. Since (τ_i) is a continuous variable:

$$\mathrm{H} = \int_0^{\infty} h^*(\tau)n(\tau)d\tau$$

where $n(\tau_i)$ is number of workers at talent τ_i in the country.

Model's solution

Follow the above settings, we can solve for total human capital stocks of the source country in cases with and without emigration, and then compare the difference between them.

Human capital formation without emigration (H_0)***:***

When there is no chance to emigrate, the worker's objective function is:

$$\underset{(c)}{Max}(TU_i) = -c_i + c_i^{\alpha}\tau_i$$

Solving the problem:

[3] For simplicity, this assumption is acceptable, and similar to "general emigration" assumption in Mountford 1997.

$$\frac{\partial(TU_i)}{\partial c_i} = -1 + \alpha c_i^{\alpha-1}\tau_i = 0$$

$$\Rightarrow c_i^* = (\alpha\tau_i)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow h_i^* = (c_i^*)^\alpha \tau_i = (\alpha)^{\frac{\alpha}{1-\alpha}} \tau_i^{\frac{1}{1-\alpha}}$$

Thus, the aggregate human capital formation of the economy without chance for emigration is:

$$\mathrm{H}_0 = \int_0^\infty h^*(\tau)n(\tau)d\tau$$

$$= N\int_0^\infty (\alpha)^{\frac{\alpha}{1-\alpha}} \tau^{\frac{1}{1-\alpha}} p(\tau)d\tau$$

or

$$\mathrm{H}_0 = N\left((\alpha)^{\frac{\alpha}{1-\alpha}} \int_0^\infty \tau^{\frac{1}{1-\alpha}} p(\tau)d\tau \right) (1)$$

Human capital formation with emigration* (H)*:

When it is possible to emigrate, the worker faces a probability π of going abroad and receiving the income $\omega(c_i^\alpha \tau_i)$, and a probability $(1-\pi)$ of staying to work in the home country and receiving an income $(c_i^\alpha \tau_i)$. Therefore, her expected income is:

$$E(U_i) = \omega(c_i^\alpha \tau_i)\pi + (c_i^\alpha \tau_i)(1-\pi)$$

Now, her objective function is:

$$\underset{(c)}{Max}(TU_i) = -c_i + \omega(c_i^\alpha \tau_i)\pi + (c_i^\alpha \tau_i)(1-\pi)$$

Solving the problem:

$$\frac{\partial(TU_i)}{\partial c_i} = -1 + \alpha c_i^{\alpha-1}\tau_i(1-\pi) + \omega\alpha c_i^{\alpha-1}\tau_i\pi = 0$$

$$\Leftrightarrow -1 + \alpha\tau_i(1+\gamma\pi)c_i^{\alpha-1} = 0$$

$$\Leftrightarrow c_{iW}^* = [\alpha\tau_i(1+\gamma\pi)]^{\frac{1}{1-\alpha}}$$

where $\gamma = (w\text{-}1) > 0$.

Thus, the human capital to be accumulated by each worker is:

$$\Rightarrow h_{iW}^* = (c_{iW}^*)^\alpha \tau_i = [\alpha(1+\gamma\pi)]^{\frac{\alpha}{1-\alpha}} \tau_i^{\frac{1}{1-\alpha}}$$

The aggregate human capital formation of the economy with chance for emigration is:

$$\mathrm{H} = \int_0^\infty h_W^*(\tau)n_W(\tau)d\tau$$

where n_{iW} is the number of worker at talent τ_i staying in the country. It is obvious that:

$$n_{iW} = (1-\pi)n_i = (1-\pi)Np(\tau_i)$$

Then,

$$\mathrm{H} = N\int_0^\infty [\alpha(1+\gamma\pi)]^{\frac{\alpha}{1-\alpha}} \tau^{\frac{1}{1-\alpha}} (1-\pi)\, p(\tau)d\tau$$

$$\mathrm{H} = (1+\gamma\pi)^{\frac{\alpha}{1-\alpha}}(1-\pi).N\left((\alpha)^{\frac{\alpha}{1-\alpha}} \int_0^\infty \tau^{\frac{1}{1-\alpha}} p(\tau)d\tau \right)$$

or:

$$\mathrm{H} = (1+\gamma\pi)^{\frac{\alpha}{1-\alpha}}(1-\pi)\mathrm{H}_0 \qquad (2)$$

(2) expresses the aggregate domestic stock of human capital (H) as a function of possibility of emigration (π): $\mathrm{H} = \mathrm{H}(\pi)$.

If $\pi = 0$: $\mathrm{H} = \mathrm{H}_0$. This is the case of no emigration.

If $\pi = 1$: $\mathrm{H} = 0$. This is the case of definitely free emigration. The economy is totally destroyed (or disappeared) because all human capital stock of the country will flow abroad where human capital income is higher.

We now consider the case $\pi \in (0,1)$. From (2) $\Rightarrow \mathrm{H} > 0 \forall \pi \in (0,1) \Rightarrow$ it is possible to take log both two sides of (2):

$$\Rightarrow \ln \mathrm{H} = \frac{\alpha}{1-\alpha}\ln(1+\gamma\pi) + \ln(1-\pi) + \ln \mathrm{H}_0$$

$$\Rightarrow \frac{\partial \ln \mathrm{H}}{\partial \pi} = \frac{\alpha}{1-\alpha}\frac{\gamma}{(1+\gamma\pi)} - \frac{1}{1-\pi}$$

$$\Rightarrow \frac{\partial \mathrm{H}}{\partial \pi} = \mathrm{H}\left(\frac{[(\gamma+1)\alpha - 1] - \gamma\pi}{(1-\alpha)(1+\gamma\pi)(1-\pi)} \right)$$

$$\text{Since } \mathrm{H}\left(\frac{1}{(1-\alpha)(1+\gamma\pi)(1-\pi)} \right) > 0 \forall \pi \in (0.1) \Rightarrow$$

$$\Rightarrow \text{ sign } \left[\frac{\partial \mathrm{H}}{\partial \pi} \right] = \text{ sign } \{[(\gamma+1)\alpha - 1] - \gamma\pi\}$$

Proposition 1: If $\alpha < \frac{1}{1+\gamma}$, the source economy always suffers from losing human capital stock (real brain drain) regardless probability of emigration π. The higher the probability is, the more the country loses its human capital stock. This situation may be called *"brain drain trap."*

Proof. $\alpha < \frac{1}{1+\gamma} \Rightarrow [(\gamma+1)\alpha - 1] < 0 \Rightarrow \{[(\gamma+1)\alpha - 1] - \gamma\pi\} < 0 \ \forall \pi > 0$

$\Rightarrow \frac{\partial \mathrm{H}}{\partial \pi} < 0 \quad \forall \pi > 0 \Rightarrow \mathrm{H} < \mathrm{H}(\pi = 0) = \mathrm{H}_0 \quad \forall \pi > 0$, and $\mathrm{H}(\pi)$ is decreasing in $\forall \pi > 0$. □

In this case, the relationship between the domestic human capital stock H and the probability of emigration π is depicted in Figure 2.

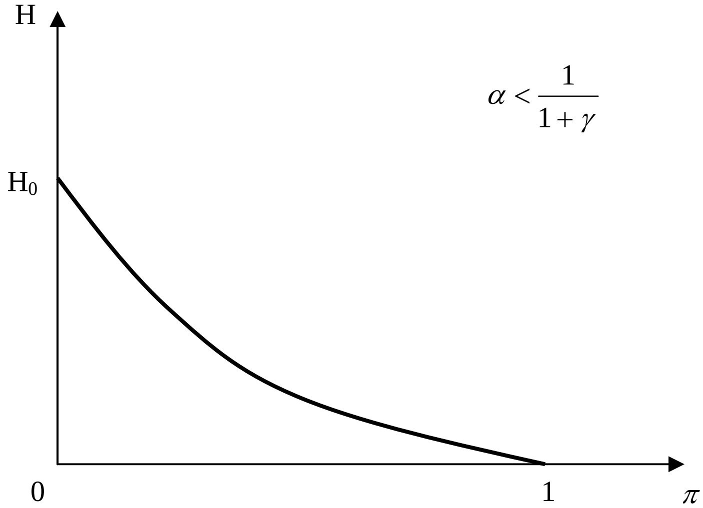

Figure 2. $\alpha < \frac{1}{1+\gamma}$: **the economy is in a "brain drain trap"**

Proposition 2: If $\alpha > \frac{1}{1+\gamma}$, there exists a critical value of emigration probability $\pi^* = \frac{[(\gamma+1)\alpha - 1]}{\gamma}$ maximizing domestic human capital formation. π^* is the probability creating *"optimal brain drain."*

Proof. $\alpha > \frac{1}{1+\gamma} \Rightarrow [(\gamma+1)\alpha - 1] > 0 \Rightarrow \exists \pi^* = \frac{[(\gamma+1)\alpha - 1]}{\gamma} > 0$ so that:

$$\{[(\gamma+1)\alpha - 1] - \gamma\pi\} > 0 \text{ if } \pi \in (0, \pi^*) \qquad \Leftrightarrow \frac{\partial \mathrm{H}}{\partial \pi} > 0 \text{ if } \pi \in (0, \pi^*)$$

$$\{[(\gamma+1)\alpha - 1] - \gamma\pi\} = 0 \text{ if } \pi = \pi^* \qquad \Leftrightarrow \frac{\partial \mathrm{H}}{\partial \pi} = 0 \text{ if } \pi = \pi^*$$

$$\{[(\gamma+1)\alpha - 1] - \gamma\pi\} < 0 \text{ if } \pi \in (\pi^*, 1) \qquad \Leftrightarrow \frac{\partial \mathrm{H}}{\partial \pi} < 0 \text{ if } \pi \in (\pi^*, 1)$$

$\Rightarrow$ H is maximized at $\mathrm{H}_M = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[\frac{(\gamma+1)^{\frac{1}{1-\alpha}}}{\gamma}\right]\mathrm{H}_0$ when $\pi = \pi^* = \frac{[(\gamma+1)\alpha - 1]}{\gamma}$. □

Behavior of the domestic human capital stock is depicted in Figure 3.

Proposition 2 shows that when the condition $\alpha > \frac{1}{1+\gamma}$ is satisfied, a small probability of emigration at first will have positive effect on aggregate human capital formation of the source country, because “brain gain” effect dominates “brain drain” effect. It is shown that there exists a critical value of emigration probability $\pi^* = \frac{[(\gamma+1)\alpha - 1]}{\gamma}$ that maximizes the net brain gain, or the aggregate domestic human formation (point M in Figure 3). If the possibility of emigration becomes higher, the net brain gain will decrease, and at a level π^{**}[4] high enough, the brain gain effect is dominated by the brain drain effect, making the total effect equal to zero. Finally, if emigration becomes certain (π=1), the economy will lose all of its human capital stock.

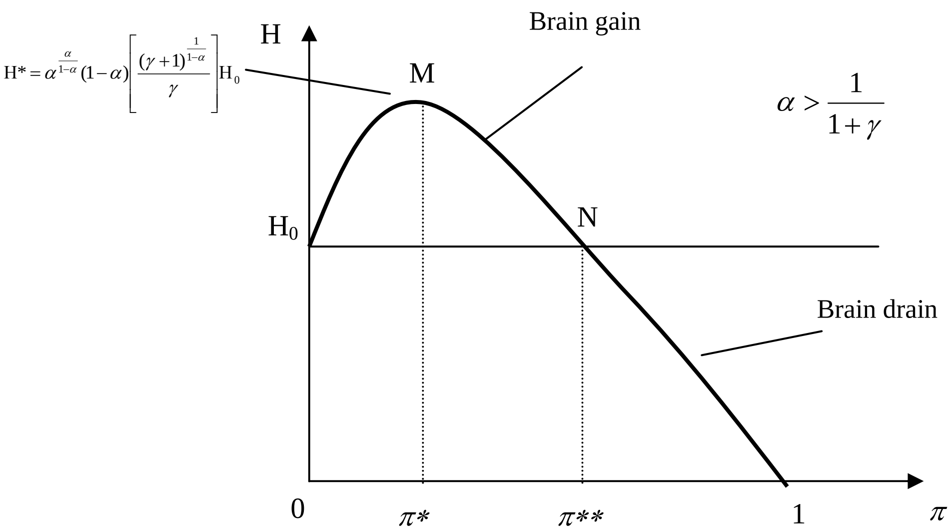

Figure 3. $\alpha > \frac{1}{1+\gamma}$**: the existence of optimal brain drain**

Policy Implications

1. From Proposition 1. In the case $\alpha < \frac{1}{\omega}$, that may be called “brain drain trap”, the model implies that:

  (a) Given the economic conditions in receiving country (or value of ω), if α is too small, the source country always loses their human capital stock. α can be understood as the source country’s degree of technology of knowledge transfer or human capital formation. The higher α is, the more productive the formation is ($h_i = c_i^{\alpha}\tau_i$). A higher α means a better education system, or some kind of social organization which

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[4] π^{**} is the solution for the problem $(1+\gamma\pi)^{\frac{\alpha}{1-\alpha}}(1-\pi) = 1$, as shown in Figure 3 at point N, where the curve $H(\pi)$ intersects the horizontal line $H = H_0$.

allow effective learning-by-doing. This condition shows the advantage in economic integration of those countries whose human capital formation capability is high.

(b) Given α , or given the domestic technology of knowledge transfer in the source countries, if the income differential between the source and the receiving countries is lower than a certain level $(1/\alpha)$, it is harmful to have emigration. In this case, the differential is not high enough to create sufficient motivation for accumulating new human capital in the source country, therefore the out-flowing human capital is always larger than the newly created human capital.

(c) Policy implications in this case: improving domestic education quality and other ways of transferring knowledge, such as learning-by-doing in workplace. When integrating into the world, wage and opportunity differential abroad will create a demand for education, but if the domestic education fails to meet the increasing demand, the country will lose its human capital.

2. From Proposition 2. When the condition $\alpha > \frac{1}{\omega}$ is satisfied, the model suggests that:

(a) A positive probability of brain drain is not always as bad as thought. It is not a zero-sum game between the source and the receiving countries. Emigration possibility motivates people in the source country to accumulate more human capital, and at a sufficient low probability of emigration, the source country can gain human capital from this process.

(b) There exists a unique value of emigration probability that maximizes the human capital gain of the source country. It is the point of optimal brain drain. The source country's government can use emigration policies to control this probability to lead the economy to the optimality. It is suggested that controlling the probability may be less costly than subsidizing domestic education in creating human capital.

(c) The model confirms that, in any case, a sufficient high value of emigration probability will damage the source country's human capital stock (real brain drain). This means that a control in emigration and brain drain is always necessary.

CONCLUSION

In this paper I have constructed a model of optimal brain drain. The model proves that under some conditions, the source country may get stuck in a "brain drain trap," where emigration possibility always leads to net brain drain, or the brain drain effect at all times dominates the brain gain effect. The conditions concern domestic quality of human capital formation technology (ie. education system or social system for learning-by-doing in workplace) and the marginal human capital product differential between the two countries. However, if the source country is not in a "brain drain trap," it is possible to accumulate human capital by allowing a certain possibility of emigration. Emigration possibility may create motivation for human capital accumulation in the source country. Consequently, the paper shows two critical points in this process: an "optimal brain drain" value of the emigration probability, where the source country is able to get maximum human capital stock; and a "net brain drain" value where the source country begins to lose its human capital.

The model proposed in this paper is a base-model. Although it is simplified in many ways and developed in a static framework, its message is basic and straightforward. For further studies, we may consider emigration probability as a function of workers' human capital $\pi_i = \pi(h_i)$ and assume a general form of human capital formation function. We may also examine the progress of human capital stock in a dynamic framework, using overlapping generation approach with human capital bequest from workers' parents. A more comprehensive study may consider externality of human capital as suggested by Lucas (1988). Moreover, to investigate the dynamics of an education sector facing increasing demand may bring interesting results. Finally, concerning education policy in open economies, we can analyze the country's welfare with different patterns of education (i.e. public versus private investment in education and training, credit with and without constraints, etc.) in presence of emigration.

REFERENCE:

Barro, Robert and Xavier Sala-I-Martin, 2004. *Economic Growth.* Cambridge, Mass: MIT Press.

Beine, Michel, Frederic Docquier, Hillel Rapoport, 2001. "Brain Drain and Economic Growth: Theory and Evidence." *Journal of Development Economics*, 64: 275-289.

Bhagwati, Jagdish N., Koichi Hamada, 1974. "The Brain Drain, International Integration of Markets for Professionals and Unemployment: A Theoretical Analysis." *Journal of Development Economics*, 1: 19-24.

Bhagwati, Jagdish N., ed., 1976. *The Brain Drain and Taxation, Vol. II: Theory and Empirical Analysis.* Amsterdam: North-Holland.

Bhagwati, Jagdish N., 1979. "International Migration of the Highly Skilled: Economics, Ethics and Taxes." *Third World Quarterly*, 1: 17-30.

Bhagwati, Jagdish N., 1979. "The Economic Analysis of International Migration." Lecture delivered to Nordic Council of Ministers, Oslo, Norway, reprinted in Bhagwati, Jagdish N., *International Factor Mobility (Essays in International Economic Theory, Vol. II.* Cambridge, Mass: MIT Press (1987).

Bhagwati, Jagdish N., Martin Partington, eds., 1976. *Taxing the Brain Drain, Vol.. I: A Proposal*. Amsterdam: North-Holland.

Hamada, Koichi, 1977. "Taxing the Brain Drain: A Global Point of View," in J. N. Bhagawati, ed., *The New International Economic Order: The North-South Debate.* Cambridge, Mass: MIT Press.

Lowell, Lindsay B., 2002b. "Some Developmental Effects of the International Migration of Highly Skilled Persons." *International Migration Papers*, No. 46, International Labour Office, Geneva.

Lucas, Robert E. Jr., 1988 "On the Mechanics of Economic Development." *Journal of Monetary Economics*, 22: 3-42.

Miyagiwa, Kaz, 1991. "Scale Economics in Education and the Brain Drain Problem." *International Economic Review*, 32: 743-759.

Mountford, Andrew, 1997. "Can a Brain Drain be Good for Growth in the Source Economy?" *Journal of Development Economics*, 53: 287-303.

Rapoport, Hillel, 2002. “Who is Afraid of the Brain Drain? Human Capital Flight and Growth in Developing Countries.” *Policy Brief*, Stanford Institute for Economic Policy Research, Stanford.

Stark, Oded, 2004. “Rethinking Brain Drain.” *World Development*, 32: 15-22.

Stark, Oded, Christian Helmenstein, Alexia Prskawetz, 1997. “A Brain Gain with a Brain Drain.” *Economics Letters*, 55: 227-234.

Stark, Oded, Christian Helmenstein, Alexia Prskawetz, 1998. “Human Capital Depletion, Human Capital Formation, and Migration: a Blessing or a “Curse”?” *Economics Letters*, 60: 363-267.

Stark, Oded, Yong Wang, 2002. “Inducing Human Capital Formation: Migration as a Substitute for Subsidies.” *Journal of Public Economics*, 86: 29-46.

Vidal, Jean-Pierre, 1998. “The Effect of Emigration on Human Capital Formation.” *Journal of Population Economics*, 11: 589-600.