Do Stock Prices in HoSTC Have Unit Root? A Discussion on Power of ADF F Test with Unexpected Initial Value

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Objectives

- Apply most powerful unit root (UR) tests for Random Walk Hypothesis (RWH) in HoSTC (Ho Chi Minh City Stock Trading Center), Vietnam.
  - Unit root being rejected implies a violation of RWH (and also Efficient Market Hypothesis (EMH), assumed $E(R_t) = R \sim$ const: $E(R_t|P_{t-1}, P_{t-2,\dots}) = E(R_t)$.
  - Acceptance of null suggests further investigation.
  - Permanent/short-term effect of shocks on the future behavior.
- Show the power superiority of Dickey and Fuller's F test (ADF-F) as well as Holden and Perman's test procedure (HP-ADF) when unexpected initial value (IV) exists.
- Recommend for the robustness of UR test.

Outline

- Investigate the power behavior of ADF-F and t test upon the change in IV
- Propose HP-ADF test procedure as an approach improving power
- Apply robust tests for the data of HoSTC and discuss on the results
- Conclusion

Brief introduction to HoSTC

- First trading day: 28th July, 2000
- Total capitalization less than 1% GDP(*)
- Daily trading value: Less than 500,000 USD/day(*).
- Room for foreigner participation: 49% (bank: 30%)
- Operation: Daily since 1st March, 2002
- Mechanism: automated order-matching system (300,000 orders/day), two matches/day(*)
- Minimum order: 10 shares (~7 USD of par value)

(*) Updated information is considerably different.

Brief introduction to HoSTC

- It is believed that the market is inefficient even in the 'weak form':
  - The market is driven by herd behavior
  - Information regulation incompliance and leakage
  - Lack of consulting and rating reports
  - Strict price limit regulation
  - Small scale market which is vulnerable to the manipulation of big traders

This paper is a first analysis concerning the weak from market efficiency for Vietnamese stock market, using unit root tests as a preliminary step

Power and size of UR tests

- The original Dickey and Fuller's ADF tests (1979 and 1981) and Philips and Perron's PP tests (1988):
  - Size distortion is significant when the moving average component has large negative root (Schwert (1989), Perron and Ng (1996)),
  - Low power if the autoregressive coefficient is close to unity (DeJong et al., 1992).
- The modified ADF test of Elliot, Rothenberg, and Stock (ERS or GLS-DF test, 1996) and the modified Z test of Perron and Ng (GLS-MZ test, 2001):
  - Improve both size and power problem above as long as the lag length is appropriately selected (e.g. by modified AIC)
  - Lose power when the nuisance parameter, initial value, is far from the deterministic trend under alternative.

Power and initial value

- The recently modified tests (ERS, 1996/1999 and GLS-MZ, 2001) lose their power for moderately large IV in absolute term
- …ADF-t test, contrarily, gains power (Muller and Elliot, 2003, Dejong et al. (1992))
- The choice of appropriate tests depends on the knowledge of initial value.
- Large initial value may occur when studying new market or institution => employ various efficient tests that are most powerful with different magnitude of IV is necessary.

Model

$$y_t = c + \alpha y_{t-1} + \beta t + \varepsilon_t, \qquad t = 1, 2, \ldots, T \qquad (1)$$

$$\varepsilon_t \sim IID(0, \sigma^2); \alpha < 1$$

c, **α**, and **β** are fixed parameters. T is finite but big enough to approximate:

$$\sum_{t=1}^{T} \alpha^{t-1} \approx 1/(1-\alpha)$$

Unconditional expectation of IV, y_0:

$$w_0 = c/(1-\alpha) - \beta\alpha/(1-\alpha)^2 \qquad (2)$$

Recursive equation for y_t, given $\Delta y_0 = y_0 - w_0$:

$$y_t = w_0 + \frac{\beta t}{1-\alpha} + \Delta y_0 \alpha^t + \sum_{i=0}^{t-1} \alpha^i \varepsilon_{t-i}$$

Deriving t statistic of ADF test

- F statistic is based on the ratio between the restricted sum of squared errors (SSE) and the unrestricted SSE
- t statistic squared is a special F statistic with only one restriction
- When Δy_0 changes, relative change of the unrestricted SSE is ignorable.
- The restricted errors is the residuals of the regression of the difference series $z_t = y_t - y_{t-1}$:

$$z_t = c + \beta t + e_t$$

Deriving t statistic of ADF test

- Transformations lead to the approximation:

$$t_{stat}^2 \approx \frac{(\Delta y_0)^2 B + \sum_{t=1}^{T} \left[I_t(\varepsilon) \right]^2 - SSE_0}{SSE_0} \times \left(T - k \right)$$

- Where B is mainly the difference between $q = \frac{1-\alpha}{1+\alpha}$ and the term $O(T^{-1})$.
- $I_t(\varepsilon)$ is the function of $\{\varepsilon_t\}$ and T
- Given $\{y_t\}$, when IV more deviates from its unconditional expectation, t^2 statistic increases implying the power improvement of ADF t test.

F statistic

- Similarly, we derive the approximation of F statistic

$$F_{stat} \approx \frac{(\Delta y_0)^2 A + \sum_{t=1}^{T} \left[H_t(\varepsilon)\right]^2 - SSE_0}{SSE_0} \times \frac{(T-k)}{2}$$

- Where $A = \frac{1-\alpha}{1+\alpha} - \frac{1}{T} = q - 1/T$

- $H_t(\varepsilon)$ is the function of $\{\varepsilon_t\}$ and T
- Given $\{y_t\}$, when IV more deviates from its unconditional expectation, *F* statistic increases implying the power improvement of ADF F test.

Comparison of F and t statistic

$$F_{stat} \approx F_0 + \frac{\left(\Delta y_0\right)^2 A}{SSE_0} \times \frac{(T-k)}{2}$$

$$t_{stat}^2 \approx t_0^2 + \frac{\left(\Delta y_0\right)^2 B}{SSE_0} \times (T-k)$$

- F_0 and t_0^2 are the statistics given $\Delta y_0 = 0$
- If T is not so large, A is considerably larger than B, F statistic grows faster making F test is potentially gain more power than t^2 test when $|\Delta y_0|$ increases.

Comparison of F and t statistic

$$K = \frac{F - F_0}{t^2 - t_0^2} = \frac{A}{2B}$$

- When K is big enough, the critical value of F test will 'move' to zero before that of t test.
- Numerical example: $\alpha = 0.98$; T = 1,000; significance level: 5%.
  - $F_{crit} = 6.25$ and $t_{crit} = -3.41$ or $t_{crit}^2 = 11.63$.
  - q = 0.02/1.98 = 0.0101, A = q – 1/T = 0.0091, B = 0.0067

$$K = \frac{0.0091}{2 * 0.0067} = 0.6827$$

  - When F_{crit} 'moves' to 0, t^2_{crit} 'moves' to t^{*2}:

$$t_{crit}^2 - t^{*2} = F_{crit} / K = 6.25 / 0.6827 = 9.16$$

$$t^{*2} = 11.63 - 9.16 = 2.47 > 0$$

Comparison of F and t statistic

- Both F- and t- tests are not affected by the nuisance parameters β and c but depend on $|\Delta y_0|$, T, and α.
- The power of (one-side) t test is higher than F test given small $|\Delta y_0|$
- The power of both tests increase as $|\Delta y_0|$ grows from certain large value
- F test becomes superior to t test when $|\Delta y_0|$ is large enough due to its higher sensitivity to $|\Delta y_0|$, not from join-test nature by itself.
- Closer is α to unity, more distinguishable is F test to t test
- When the sample size increases, both tests have the power approaching unity and the difference between them gradually reduces

ADAPTED HOLDEN AND PERMAN'S PROCEDURE (1994)

- Step 1: Estimate the equation:

$$y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{l-1} \zeta_i \Delta_{y_{t-1}} + \varepsilon_t$$

- Step 2: Use $\boldsymbol{\Phi}_3$ statistic to test: H_0: (**α** , **β** , **ρ**)=(**α** ,0,1) versus H_1: (**α** , **β** , **ρ**)≠(**α** ,0,1). If null is rejected, go to step 3, otherwise, go to step 5
- Step 3: Test **ρ** = 1 using the t-statistic from step 1, with critical values from the standard normal tables.
  - If the null is not rejected, conclude: process has unit root with time trend (rare).
  - Otherwise, the process is stationary with/without time trend

ADAPTED HOLDEN AND PERMAN'S PROCEDURE (1994) - continued

- Step 5: Use a t statistic to test for $\boldsymbol{\rho} = 1$, assuming $\boldsymbol{\beta}$ is zero – non-standard critical values are required (t test is expected to consistent with F test, $\boldsymbol{\Phi}_3$)
- The further steps (4, 6, and 7) are not used because:
  - $\boldsymbol{\Phi}$1 and $\boldsymbol{\Phi}$2 are proved to be redundant (John Elder and Peter E. Kennedy, 2001).
  - Drift and existence of trend are not of primary concerns.

Holden and Perman's approach

- HP-ADF basically has similar size to t and F tests in relevant cases.
- HP-ADF is more powerful than ADF t test when $|\Delta y_0|$ is large enough
- Following the additional rule that: if F test cannot reject null while t test can, we reject the null, HP-ADF would be more powerful than F test when $|\Delta y_0|$ is moderate
- HP-ADF avoid over rejection of F test when the model is unit root with time trend.
- In sum, HP-ADF is superior to both F and t test alone in the appropriate situation, especially, in the case of large $|\Delta y_0|$, HP-ADF is more powerful than t test.

Simulation model

$$y_t = \alpha y_{t-1} + \beta t + \varepsilon_t$$

$$\varepsilon_t \sim IIN(0, 0.01), \quad t = 1, 2, \ldots T$$

$$T = 500 \text{ or } T = 1,000$$

$$y_0 = w_0 + \Delta y_0$$

$$w_0 = -\beta\alpha / (1-\alpha)^2$$

$$\left|\Delta y_0\right| = (0, 10, 20, \ldots, 50)\sigma$$

$$\alpha = 0.98; \beta = 0.00005$$

Replications number: 5,000 for each value of Δy_0

Tests used: HP-ADF/F, ADF test, GLS-DF

Table 1 – Power of HP-ADF, t, and GLS-DF tests with different IV[b]

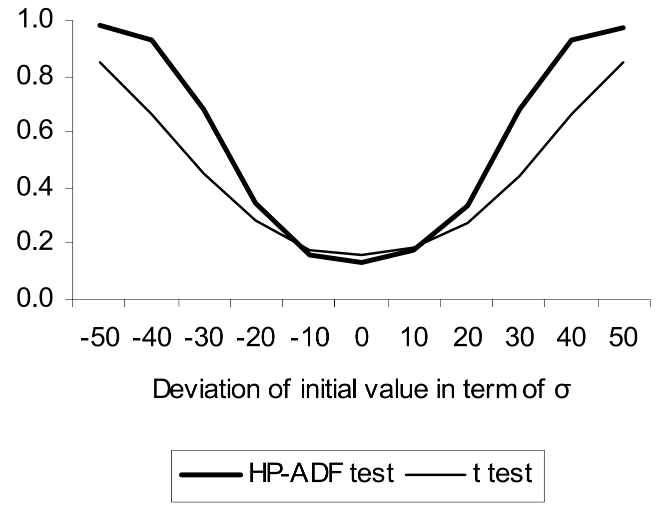
y_0	T = 500			T = 1,000		
	HP-ADF	ADF t	GLS-DF	HP-ADF	ADF t	GLS-DF
-0.6225	0.979	0.853	0.000	0.999	0.997	0.000
-0.5225	0.929	0.667	0.000	0.995	0.986	0.000
-0.4225	0.682	0.449	0.000	0.955	0.921	0.000
-0.3225	0.349	0.282	0.001	0.784	0.769	0.000
-0.2225	0.162	0.181	0.074	0.571	0.624	0.194
-0.1225[a]	0.129	0.163	0.244	0.489	0.563	0.778
-0.0225	0.177	0.189	0.064	0.575	0.632	0.194
0.0775	0.337	0.273	0.002	0.790	0.782	0.001
0.1775	0.680	0.444	0.000	0.959	0.934	0.000
0.2775	0.925	0.662	0.000	0.992	0.983	0.000
0.3775	0.976	0.848	0.000	0.998	0.996	0.000

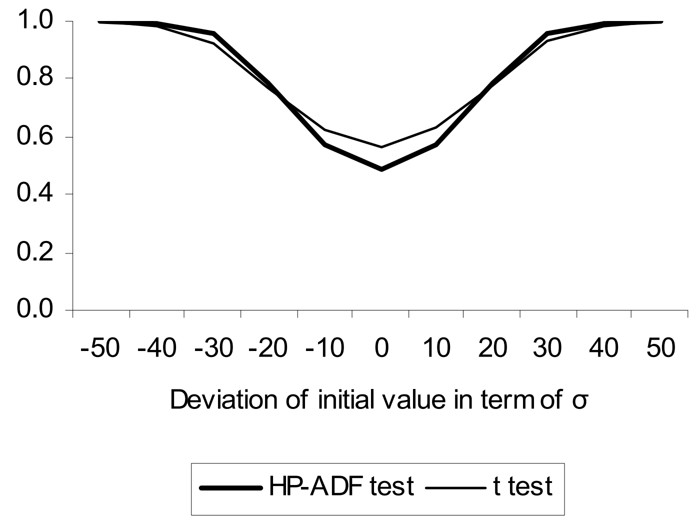
[a] The unconditional expectation of IV

[b] GLS-DF is used with MAIC for lag selection while GTS approach is used for HP-ADF and t tests.

Results

- The results are consistent with the theoretical inferences
- Both t test and F/ADF-HP test have minimum power at zero Δy_0 (Figure on the right)
- At zero Δy_0 (for both sample sizes):
  - **GLS-DF**>>t>F/HP-ADF tests
  - 24% >> 16% > 13%, T = 500
  - 78% >> 56% > 49%, T = 1,000
- At non-zero, e.g. $\Delta y_0 = 30\sigma$:
  - GLS-DF << t < **F/HP-ADF**
  - 0% << 45% < 68%, T = 500
  - 0% << 92% < 96%, T = 1,000
- The difference between HP-ADF/F and t tests diminishes when T increases
- GLS-DF loses its power very fast when IV more deviates from expected value





A remark when the errors are serial correlated

- HP-ADF test is still more powerful than ADF-t tests for small or moderate sample sizes
- Positive large AR(1) and MA(1) coefficients may diminish the superiority of HP-ADF/F test over ADF t test
- Size distortion of both tests are similar:
  - Considerable size distortion when MA(1) coefficient is negative
  - Acceptable size distortion for the remaining cases except when the sample size is small, e.g. T = 100.

Robust UR for stock prices series in HoSTC and data

- Stock prices in HoSTC:
  - The most early-quoted stocks experienced the first peak in 2001 when VNINDEX achieve 571 points from the starting value of 100 points
  - Many stocks have the ‘outlier’ (high) opening prices following by adjusted periods
- Selected tests:
  - GLS-DF would improve size and power for the case of expected IV.
  - HP-ADF and ADF t tests would improve power when IV deviates far from the deterministic trend
  - KPSS testing for the null of stationary would provide a good complimentary view.
- Data: All available stock prices series of HoSTC from June 25, 2001 (the first peak of VNINDEX) to November 14, 2005:
  - 31 series (including VNINDEX) with lengths vary from 83-899.
  - The series starting before March 1st, 2002 are the alternative-day series.
  - The prices series are adjusted for dividend payments and splits and then transformed into natural logarithm form.

Examples of UR tests for stock prices series

- Supporting evidences include:
  - Panagiotidis (2004) testing for three indices of Athens Stock Exchange
  - Chan et al. (1997) testing for monthly price indices of 18 developed countries
- Rejection evidences include:
  - Abeysekera (2001) testing for SSI and FSI of Sri Lanka's indices
  - Li et al. (2002) testing for NZSE's indices (New Zealand)

Table 2 – Unit root tests for HoSTC

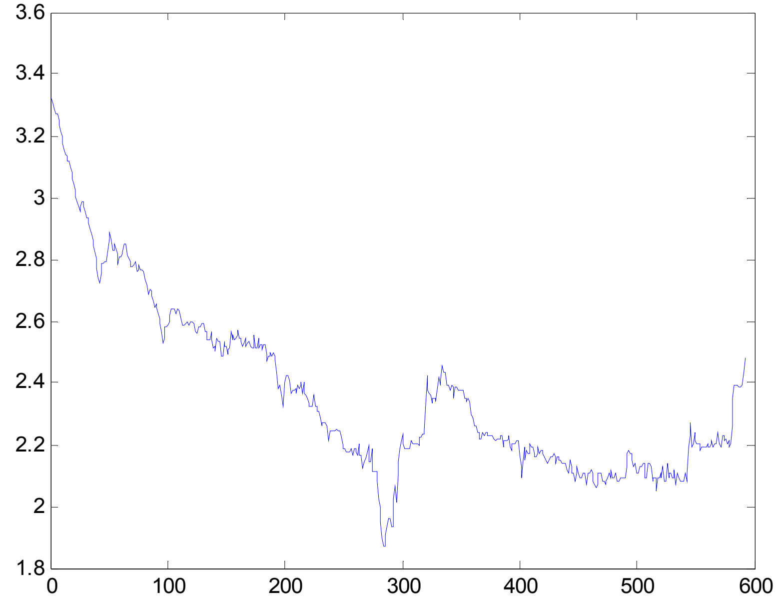
Stock	Size	DF-GLS (MAIC)		KPSS	HP-ADF				
		$t_{stat.}$	$T_{crit.}$ 5%		$t_{stat.}$	$t_{crit.}$ 5%	$t_{crit.}$ 1%	$F_{stat.}$	$F_{crit.}$ 5%
AGF	887	-1.35	-2.89	0.30**	-2.07	-3.41	-3.96	2.78	6.25
BBC	583	-0.56	-2.89	0.61**	-2.11	-3.41	-3.96	3.75	6.25
BBT	423	-0.43	-2.92	0.59**	-2.32	-3.42	-3.98	6.39*	6.31
BPC	899	-1.42	-2.89	0.30**	-2.21	-3.41	-3.96	2.82	6.25
BT6	894	-0.96	-2.89	0.35**	-2.48	-3.41	-3.96	3.55	6.25
BTC	569	-1.59	-2.89	0.37**	-2.11	-3.41	-3.97	2.85	6.27
CAN	608	-1.29	-2.89	0.58**	-0.98	-3.41	-3.96	1.66	6.25
DHA	401	-0.31	-2.89	0.46**	-1.93	-3.42	-3.98	8.24*	6.32
DPC	592	0.13	-2.89	0.55**	-1.90	-3.41	-3.96	7.64*	6.25
GIL	577	-1.42	-2.89	0.26**	-1.13	-3.41	-3.96	1.10	6.26
GMD	892	-0.77	-2.89	0.38**	-1.63	-3.41	-3.96	2.66	6.25
HAP	658	-0.62	-2.89	0.61**	-2.80	-3.41	-3.96	6.24	6.25
HAS	723	-1.56	-2.89	0.39**	-1.74	-3.41	-3.96	1.67	6.25
KHA	810	-1.45	-2.89	0.44**	-2.27	-3.41	-3.96	2.74	6.25
LAF	658	-0.50	-2.89	0.64**	-2.45	-3.41	-3.96	5.35	6.25
MHC	168	-1.78	-2.96	0.09	-4.83**	-3.44	-4.02	12.05*	6.42
NKD	230	-0.98	-2.93	0.41**	-0.10	-3.43	-4.00	4.45	6.37
PMS	509	-1.47	-2.89	0.17*	-3.44*	-3.42	-3.98	6.64*	6.30
PNC	90	-2.63	-3.07	0.23**	-3.12	-3.46	-4.06	4.94	6.54
REE	658	-0.47	-2.89	0.53**	-2.72	-3.41	-3.96	5.40	6.25
SAM	658	-0.78	-2.89	0.46**	-3.67*	-3.41	-3.96	8.23*	6.25
SAV	882	-1.17	-2.89	0.32**	-2.35	-3.41	-3.96	3.29	6.25

Table 2 – Unit root tests for HoSTC (continued)

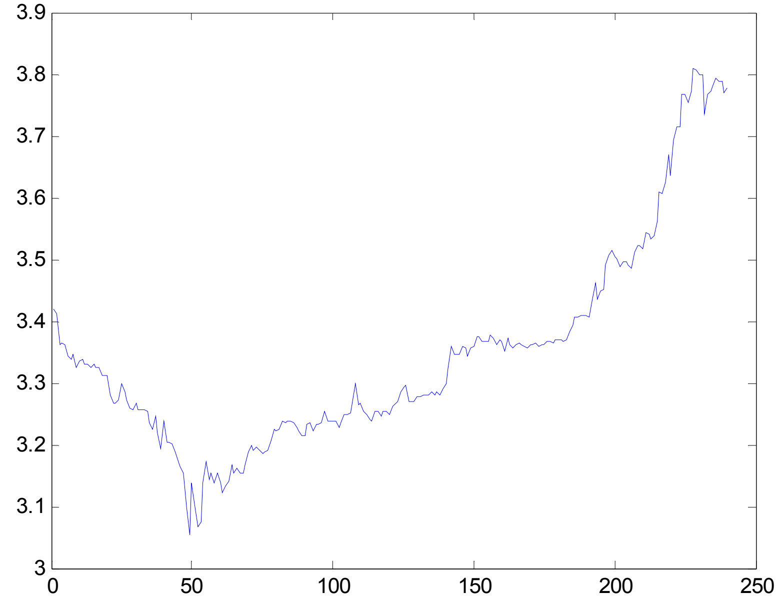
Stock	Size	DF-GLS (MAIC)		KPSS	HP-ADF				
		$t_{stat.}$	$T_{crit.}$ 5%		$t_{stat.}$	$t_{crit.}$ 5%	$t_{crit.}$ 1%	$F_{stat.}$	$F_{crit.}$ 5%
SFC	291	-0.74	-2.91	0.35**	-4.69**	-3.43	-3.99	13.02*	6.33
SGH	649	-2.12	-2.89	0.62**	-1.90	-3.41	-3.96	4.70	6.25
SSC	182	-1.67	-2.95	0.36**	-1.96	-3.44	-4.01	1.92	6.41
TMS	658	-0.71	-2.89	0.56**	-3.64*	-3.41	-3.96	9.43*	6.25
TNA	83	-1.02	-3.11	0.30**	-5.17**	-3.47	-4.08	27.28*	6.58
TRI	579	-0.93	-2.89	0.38**	-1.27	-3.41	-3.96	2.08	6.26
TS4	817	-0.97	-2.89	0.30**	-3.08	-3.41	-3.96	5.74	6.25
VNI[a]	658	-0.61	-2.89	0.55**	-3.27	-3.41	-3.96	7.84*	6.25
VTC	693	-1.31	-2.89	0.51**	-1.09	-3.41	-3.96	0.91	6.25

Note:
-For KPSS tests, critical values are 0.216 (1%), 0.146 (5%), and 0.119 (10%).
-All the tests are specified with time trend. GLS-DF is used with MAIC for lag selection. KPSS is used with Newey-West using Bartlett kernel method for selection of bandwidth. HP-ADF is used with OLS F-test and t-test for lag selections (GTS approach)
-(*) or (**) mean the tests reject null at 5% or 1% respectively.
-t critical values for HP-ADF tests shown in this table based on non-standard distribution assuming that β (time trend coefficient) is zero. If β is non-zero, t statistic would follow standard normal distribution (-1.65 at 5%, one-side).
-[a]: Stands for VNINDEX

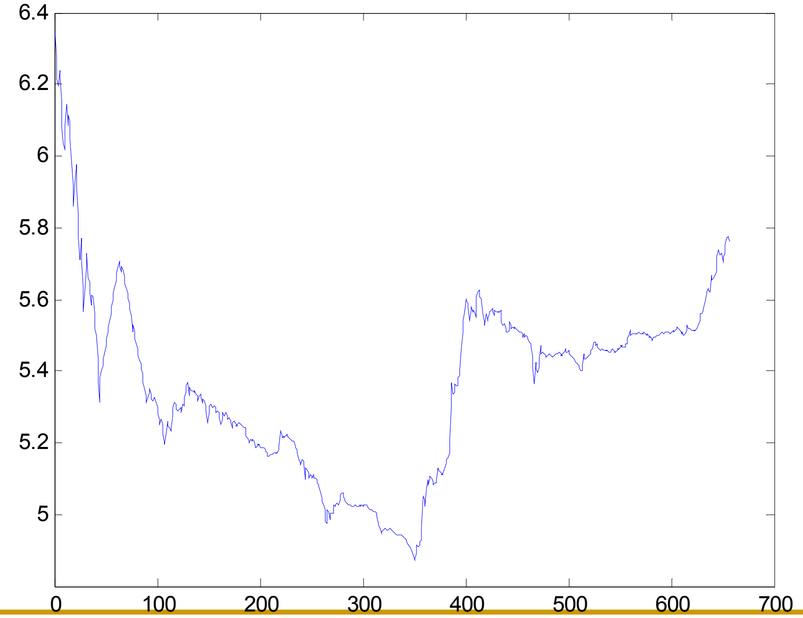
SOME REJECTED SERIES QUOTING DURING THE TESTED PERIOD



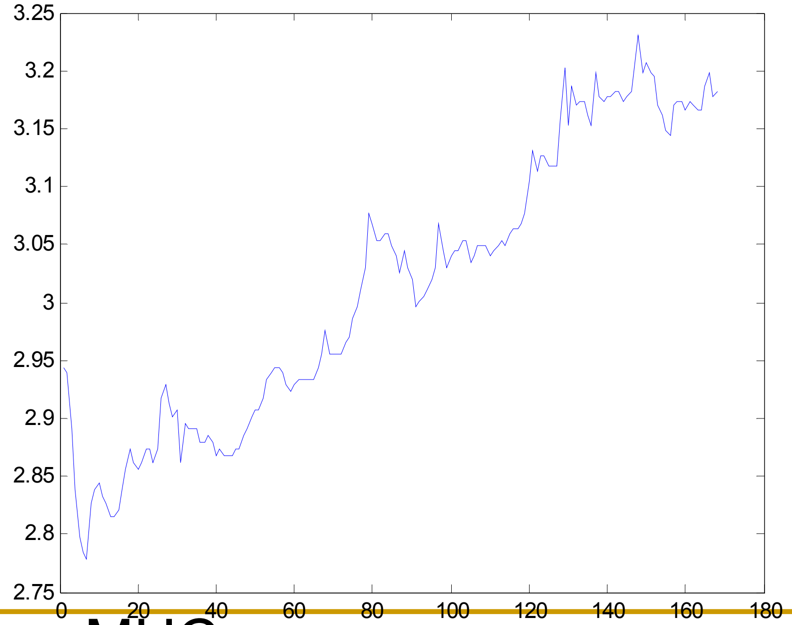
DPC



DHA



VNINDEX



MHC

Table 3 – ARMA(1,1) estimates of difference series rejected by ADF t or HP-ADF tests

Stock	Const	AR(1)	MA(1)
BBT	0.0010	0.98	-0.99
DHA	0.0009	-0.33	0.36
DPC	-0.0014	-0.34	0.42
MHC	0.0040	0.47	-0.62
PMS	0.0012	0.03	-0.23
SAM	0.0005	0.20	0.06
SFC	0.0037	0.89	-1.00
TMS	0.0016	0.97	-0.96
TNA	0.0021	0.20	0.47
VNINDEX	-0.0008	0.10	0.23

Results and discussions

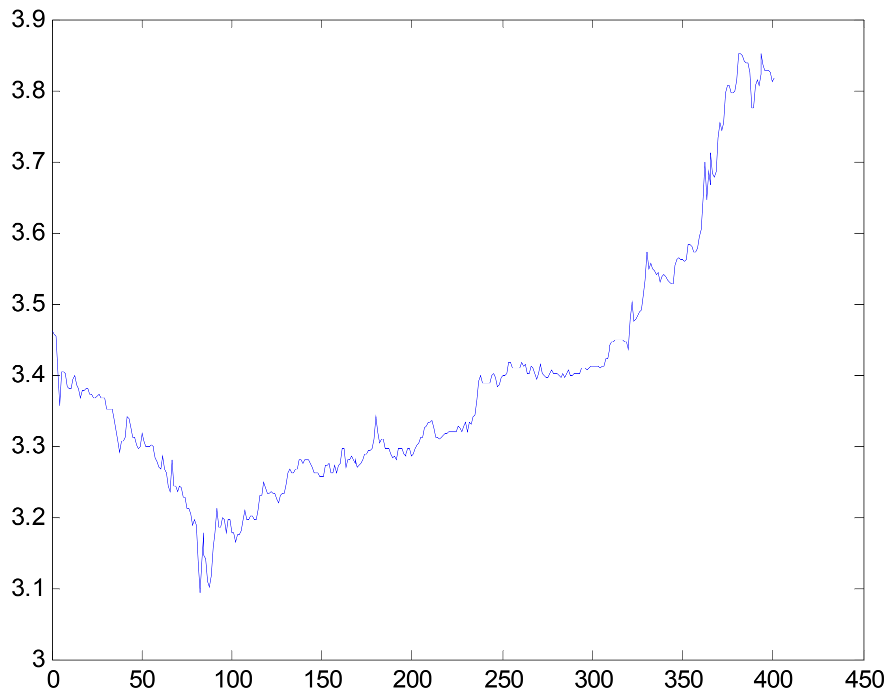
- GLS-DF and KPSS support the existence of UR for all of the series (except KPSS cannot reject null of stationarity of one series)
- All of the series which seem to have extreme IVs are rejected by HP-ADF. The rejections seem not to be caused by size distortions.
- ADF t test also rejects the series with large IVs but less powerful than HP-ADF.

⇓

The series is likely to follow AR(1) process with the root very close to unity

THE CASE OF DHA

- MA coefficient: 0.36; AR coefficient: -0.33
- ERS and KPSS are in favor of unit root.
- ADF t test cannot reject null of UR: $t_{statistic} = -1.93 > t_{critical} = -3.42$
- HP-ADF reject null of UR:
  - Φ_3 reject $(\alpha, \beta)=(0,0)$: $F_{statistic} = 8.24 > F_{Critical} = 6.31$
  - $t_{statistic} = -1.93 < -1.65 = t_{critical}$ – reject $\alpha=0$
- **DHA maybe a stationary process with unexpected initial value and time trend**



A SIMULATION OF DHA

$$y_t = 0.0624 + 0.00005t + 0.98y_{t-1} + u_t$$

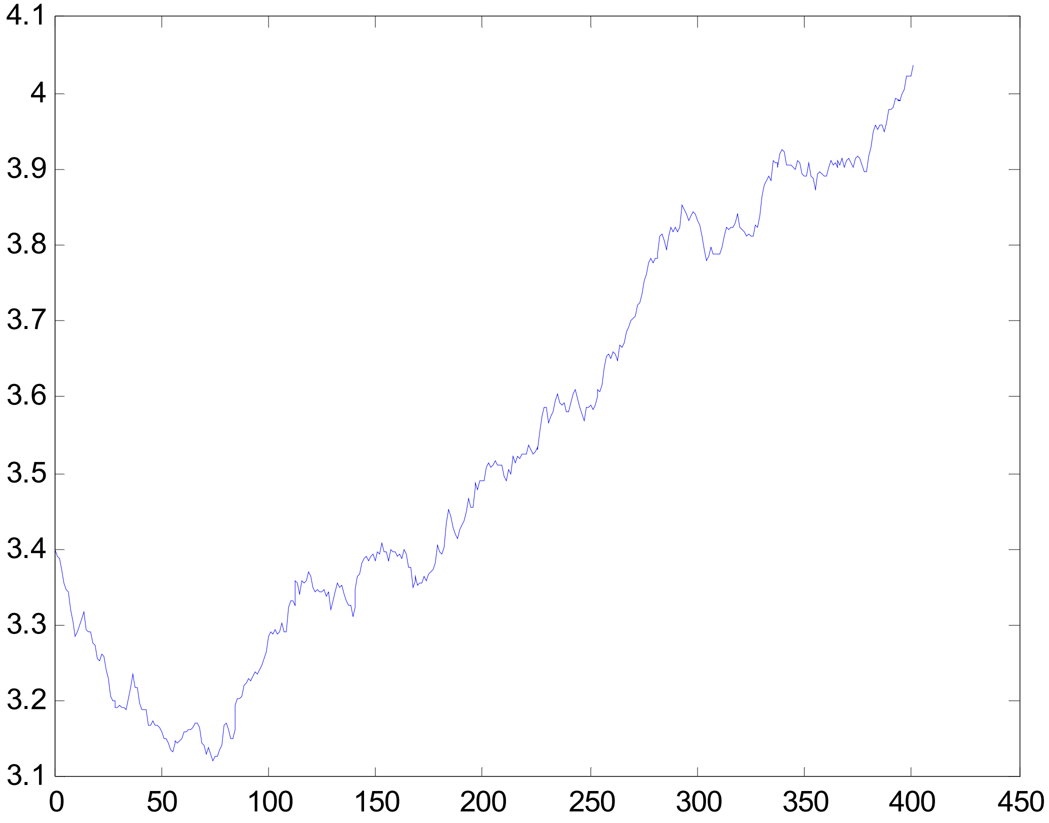
$$u_t = -0.33u_{t-1} + \varepsilon_t + 0.36\varepsilon_{t-1}$$

- $\boldsymbol{\varepsilon}_t$ ~ IIN(0,0.01), t=1,2,...,401.
- y_0 = 3.4 (real value of 3.46)

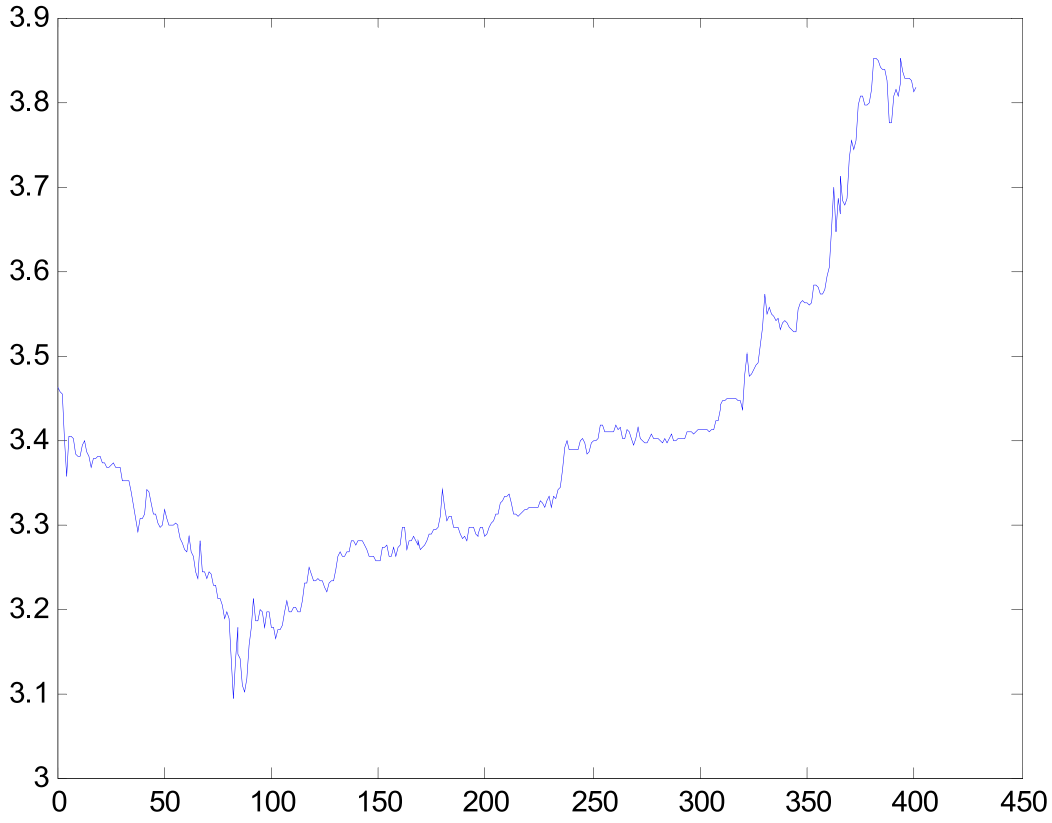
A SIMULATION OF DHA

- HP-ADF rejects 86.4% in 250 simulated series
- ADF t test rejects 43.2%
- DF-GLS (ERS-MAIC) rejects 0%
- KPSS accept null of stationarity at rate of 2/250 = 0.8%
- DHA seem to be a particular case that HP-ADF test is more powerful than DF-GLS as well as KPSS tests

CHARTS OF SIMULATED AND REAL SERIES OF DHA



A simulated series



Real series of DHA

An interpretation

- In ‘normal’ condition, stocks prices follow a process very close to random walk (or at least, UR) process, swinging about a trend
- Whenever there is a large shock, mean-reversion behavior appears; the series gradually converge to the deterministic trend. Possible causes:
  - Over-reaction and correction of the market
  - The effect of price limit regulation which does not allow the daily price change being more than 5% (the limit varied among 2%, 3%, and 7% depending on each period before set at 5% recently)

Conclusion

- The robust unit root tests surprisingly cannot decisively reject the random walk hypothesis:
  - VNINDEX seems to be more close to RW process than some indices as the ones of New Zealand and Sri Lanka markets.
- The consistent rejections of HP-ADF test for the series seemingly having unexpected initial values reflect mean-reversion behavior (including VNINDEX). This support the view that the market is inefficient even in weak-form.
- HP-ADF is strongly recommended when unexpected IV is highly possible, e.g. the case of stock prices in emerging market, besides other efficient tests like DF-GLS (Elliot et al., 1996/1999) and GLS-MZ (Ng, S., 2001)

Thank you very much for your attention!

Your questions and comments are very welcomed!